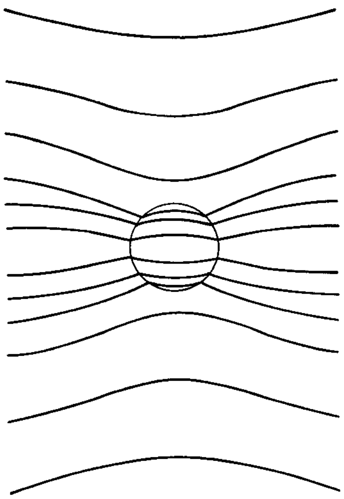

Fig. 4. Complete field mapping for HE_{11} mode. $a/\lambda_0=0.1$, $E_r=12.0$.

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On the Reflection of Waves by a Sinusoidally Stratified Half-Space

In this correspondence electromagnetic waves are taken to be obliquely incident on a sinusoidally stratified half-space. Both horizontally and vertically polarized waves are considered, and approximate reflection coefficients are obtained by using some formulas given by Heading [7].

A subject which has received considerable attention recently [1]–[6] is electromagnetic wave propagation in a sinusoidally stratified medium. This topic has application, for example, to the theory of electromagnetic waves in a plasma through which acoustic waves are propagating. In a planar stratified medium, the electromagnetic field can be expressed as the sum of two partial fields which propagate independently. These are often referred to as horizontally and vertically polarized waves, in which the electric and magnetic vectors, respectively, are parallel to the stratifications. Heading [7] studied the reflection of electromagnetic waves in a planar stratified medium by expressing the reflected field as an integral over contributions scattered back from elementary layers of thickness δz situated at level z in the medium. Approximate expressions for the reflection coefficients were obtained by using the Born approximation, thus neglecting multiple scattering. In this correspondence, Heading's single scattering formulas are applied to the problem of reflection from a sinusoidally stratified half-space.

Suppose that the region $z<0$ is a homogeneous dielectric of permittivity ϵ. The region $z>0$ is taken to have permittivity $\epsilon_r(z)$ relative to that of the medium in $z<0$, where

$$\epsilon_r(z) = \bar{\epsilon}_r[1 - A\cos(2\pi z/d + \phi)]. \quad (1)$$

In this equation $\bar{\epsilon}_r$, A, d, and ϕ are independent of z. The permeability μ is taken to be the same for all z. Losses are neglected so that the permittivity and permeability are everywhere real. Suppose that electromagnetic waves are obliquely incident from the region $z<0$. The configuration is shown in Fig. 1. Rectangular Cartesian coordinates x, y, z are used with the fields independent of y. Let E_x and H_x denote the x-components of electric and magnetic fields, with similar notation for the other components. In horizontally and vertically polarized waves the nonzero components are E_y, H_x, and H_z in the former, and H_y, E_x, and E_z in the latter.

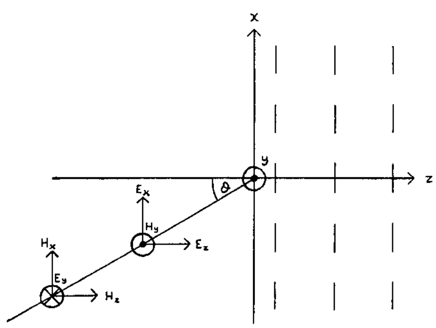

Fig. 1. The configuration.

Let the reflection coefficients for horizontally and vertically polarized incident waves be R_h and R_v, respectively. Heading's single scattering results are

$$R_h \doteq \frac{ik}{2C}\int_0^\infty [1 - \epsilon_r(z)]e^{-i2kCz}dz \quad (2)$$

and

$$R_v \doteq \frac{ik}{2C}(2C^2 - 1)\int_0^\infty [1 - \epsilon_r(z)] \cdot e^{-i2kCz}dz. \quad (3)$$

These formulas are applicable for a time factor $e^{i\omega t}$, where ω is the angular frequency and t the time. In them $k=\omega(\mu\epsilon)^{1/2}$ and $C=\cos\theta$, θ being the angle of incidence. Equations (2) and (3) are expected to be useful approximations when the resulting reflection coefficients are of small magnitude. They apply in the case of weak scattering so that $\bar{\epsilon}_r$ should be near unity and A should be small.

Equation (2) will now be evaluated, $\epsilon_r(z)$ being given by (1). Use is made of the integral

$$\int e^{ax}\cos(bx + c)dx = \frac{e^{ax}}{a^2 + b^2}[a\cos(bx + c) + b\sin(bx + c)]. \quad (4)$$

In evaluating the integral in (2) at the upper limit, it is assumed that k has a small negative imaginary part which is later allowed to tend to zero. The result is

$$R_h \doteq \frac{1 - \bar{\epsilon}_r}{4C^2} + \frac{\bar{\epsilon}_r A}{(kCd/\pi)^2 - 1}\left(\frac{kd}{2\pi}\right)^2 \cdot \left(\cos\phi + \frac{i\pi}{kCd}\sin\phi\right). \quad (5)$$

The term $(1-\bar{\epsilon}_r)/4C^2$ is the approximate reflection coefficient when the medium in $z>0$ is homogeneous and represented by $\bar{\epsilon}_r$. Thus, this term can be regarded as the reflection coefficient for an "averaged medium." The rest of (5) allows for the effects of the modulation of the permittivity about its average value.

Suppose now that $\bar{\epsilon}_r=1$. That is, the average permittivity of the half-space $z>0$ is equal to the permittivity of the half-space $z<0$. If ϕ is now taken to have the values 0 or $\pi/2$, (5) reduces to

$$R_h \doteq \frac{A(kd/2\pi)^2}{(kCd/\pi)^2 - 1} \quad (6)$$

and

$$R_h \doteq \frac{(iA/2C)(kd/2\pi)}{(kCd/\pi)^2 - 1}, \quad (7)$$

respectively. These are equivalent to (53) and (55) of Tamir et al. [1]. In these two cases, the change in permittivity across the boundary $z=0$ is a maximum (for fixed A) and zero, respectively.

It has been pointed out by a reviewer that the reflection coefficients will not be valid for wavelengths satisfying the Bragg condition. With $\bar{\epsilon}_r$ near unity and A small the Bragg condition is

$$kCd \doteq n\pi \qquad n = 1, 2, 3, \cdots. \quad (8)$$

In particular, when the first ($n=1$) Bragg condition is satisfied, (5)–(7) become infinite.

The work of Tamir et al. [1] was restricted to the case of horizontally polarized waves. Then the fields in the sinusoidally stratified medium can be expressed exactly in terms of solutions to Mathieu's equation. Particular attention was paid to the situation in which the permittivity modulations are small [1] and results were obtained by approximating the exact solutions. The case of vertically polarized waves is more complicated; the differential equation governing the fields in the sinusoidally stratified medium is Hill's equation. A series solution was obtained by Yeh et al. [3]. It is of particular interest to note that, for the reflection problem considered here, Heading's formulas give results for both horizontally and vertically polarized waves. A

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comparison of (2) and (3) shows that results for vertically polarized waves are obtained from (5)–(7) on multiplying the right-hand sides by $(2C^2-1)$.

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Y-Junction Circulator at 258 GHz

We have developed a three-port circulator at 258 GHz as part of an amplifier operating on the principle of resonance saturation of a dipolar gas.[1]

The circulator consists of a symmetric *Y*-junction with an accurately centered ferrite post. The groove dimension corresponds to RG137 rectangular waveguide (0.043 inch by 0.0215 inch). This geometry was found to have at the given frequency the least loss. The design is similar to circulators obtained by Thaxter and Heller[2] at 140 GHz.

Constructional features are a split-machined structure, optical polished and gold plated surfaces, and integrated flanges. Figure 1 shows the waveguide grooves milled in a tellurium copper block which mates with a flat counterpiece. The flat section has a center hole for a copper sleeve containing the ferrite post.

First with all three arms matched[3] the power division in the junction without tuning sleeve and ferrite was measured and found to be −4.8 dB (port 1–2) and −4.6 dB (port 1–3) for a center hole diameter of 0.024 inch. Theoretically a reflection coefficient of 0.45 was estimated from data taken at lower frequencies.[4]

Inserting the tight fitting copper sleeve with the ferrite into the junction, we experimentally determined the condition under which good circulator operation occurred. The variables are: a) the diameter of the ferrite rod and copper sleeve, b) the length the copper sleeve protrudes in the junction L_1, c) the length the ferrite rod protrudes from the copper sleeve L_2, and d) the total length of the ferrite. The results are given in Table I.

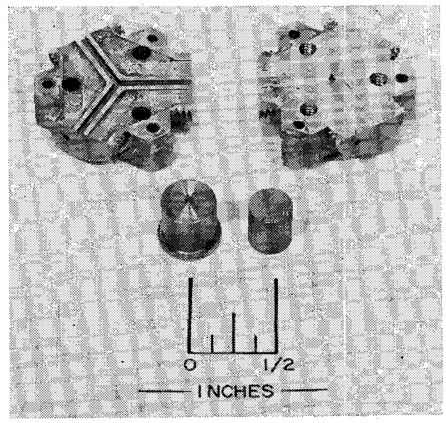

Fig. 1. Photograph of machined circulator. Copper cylinder with tuning sleeve holding ferrite magnet and the alnico magnet are shown in front.

TABLE I

Circulator number	Copper sleeve: Outer diameter (inch)	Copper sleeve: Protruding length L_1 (inch)	Ferrite (Trans Tech TT2-111): Diameter (inch)	Ferrite: Protruding length L_2 (inch)	Ferrite: Ferrite rod length (inch)	Bias field
1	0.024	0.004	0.019	0.0068	0.085	
2	0.024	0.002	0.010	0.0138	0.065	300–500 G
3	0.020	0.006	0.010	0.0140	0.065	

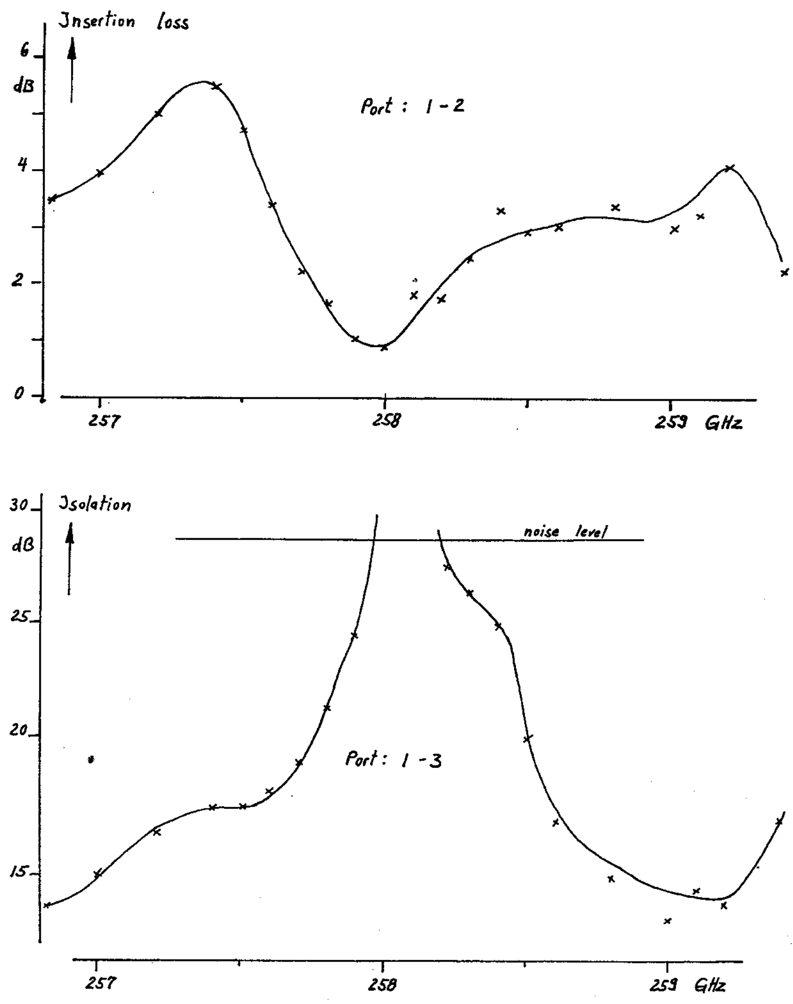

Fig. 2. Frequency characteristic of 258-GHz circulator.

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